

## Characteristics of Linear Functions (pp. 1 of 8)

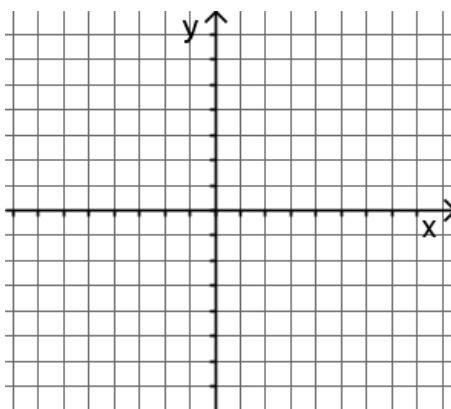
### Parent Function

Linear Parent Function:

$y =$

Domain:

Range:



Table

x	y

- What patterns do you observe in the table and graph of the linear parent function?

### Review: Characteristics

Characteristics of linear functions (such as slope, intercepts, and equations) have been addressed in previous math courses.

### Forms of Linear Equations

Standard Form:

Point-Slope Form:

Slope-Intercept Form:


### Slope-Intercept Form

In the slope-intercept form, explain how changes in  $m$  (the slope) and  $b$  (the  $y$ -intercept) affect the graph of the parent function.

Changes caused by $m$	Changes caused by $b$

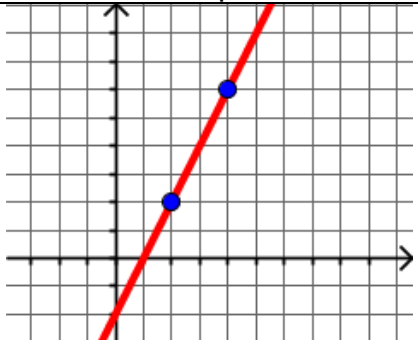
## Characteristics of Linear Functions (pp. 2 of 8)

### Finding Slope

How would you define slope?

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Explain how to find *slope* from each representation.

Graph	Table	Two Points	Equation								
	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr> <td>2</td> <td>5</td> </tr> <tr> <td>4</td> <td>19</td> </tr> <tr> <td>7</td> <td>40</td> </tr> </tbody> </table>	$x$	$y$	2	5	4	19	7	40	<p>(-1,3)</p> <p>and</p> <p>(4, -5)</p>	<p>A) <math>y = -2x + 4</math></p> <p>B) <math>y = -5 + \frac{2}{3}x</math></p> <p>C) <math>3x + y = 8</math></p>
$x$	$y$										
2	5										
4	19										
7	40										
Count	Find	Compute	Look								

### Finding Intercepts

What is an intercept?

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Explain how to find a **y-intercept** from each representation.

From the graph:	Determine where the line would cross the _____
From a table:	Look for the point _____
From the $y = mx + b$ equation:	Identify the _____
From any equation:	Plug in _____, then solve for _____

Explain how to find an **x-intercept** from each representation.

From the graph:	Determine where the line would cross the _____
From a table:	Look for the point _____
From an equation:	Plug in _____, then solve for _____

## Characteristics of Linear Functions (pp. 3 of 8)

### Finding Equations

There are several ways to determine the equation of a line, depending on the given information.

If you have...	Then plug values into...
The slope and y-intercept	
The slope, and coordinates for a point	
Coordinates for two points	
A table of data	

### Special Lines and Slopes

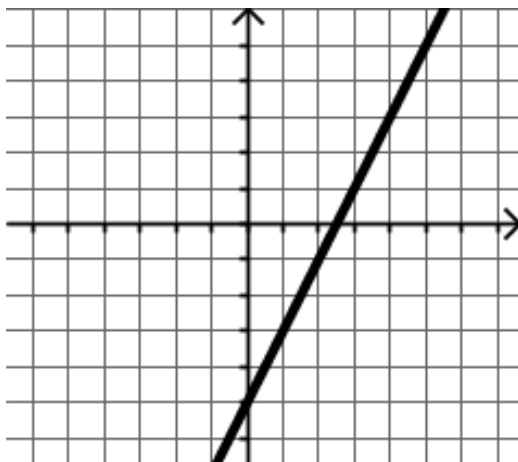
Lines	Slopes	Sample Equation(s)
Horizontal Lines		
Vertical Lines		
Parallel Lines		
Perpendicular Lines		

### Sample Problems

Provide information about each linear function.

1.

x	y

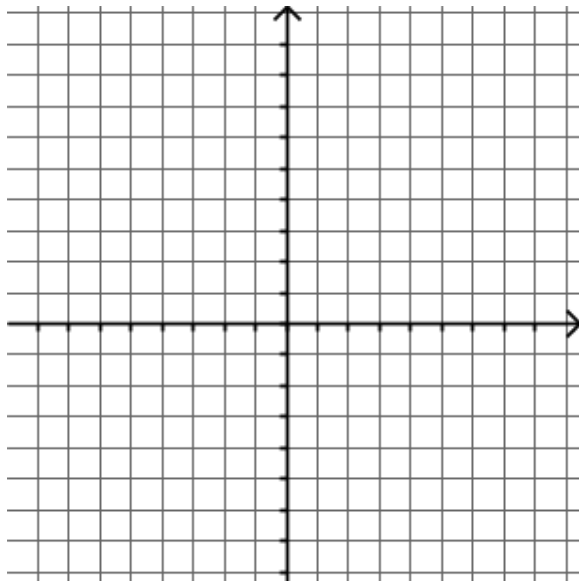


- A) Fill in the table of values.
- B) Determine the slope.
- C) Find the equation for the line.
- D) What are the intercepts?

## Characteristics of Linear Functions (pp. 4 of 8)

2.

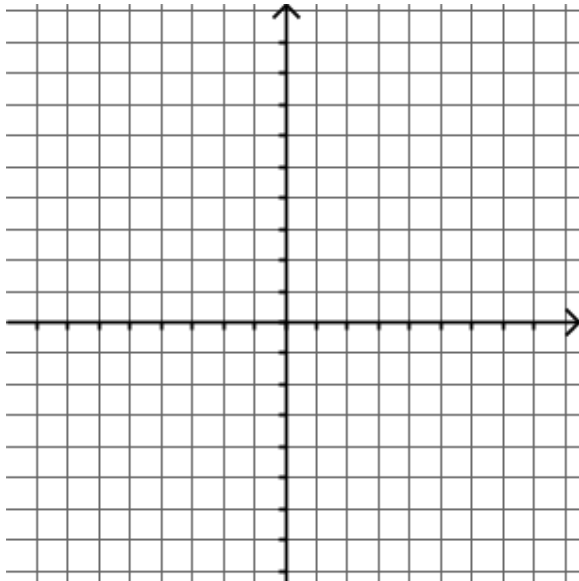
$x$	$y$
-1	10
1	4
3	-2
5	-8



- A) Graph the function.
- B) Determine the slope.
- C) Find the equation for the line.
- D) What are the intercepts?

3.

$x$	$y$



Equation:  $y = -\frac{2}{3}x + 4$

- A) Complete a table of values.
- B) Graph the function.
- C) Write down the slope.
- D) What are the intercepts?

## Characteristics of Linear Functions (pp. 5 of 8)

From the given information, find the equation of the line both in slope-intercept and standard forms.

4. The slope of  $-\frac{1}{2}$  and contains point  $(-2, 5)$

5. Contains points  $(2, -3)$  and  $(-6, 1)$

6. Contains point  $(0, 4)$  and is parallel to  $y = 2x - 3$

7. Contains point  $(-4, 5)$  and is perpendicular to  $2x + 3y = 7$

8. Contains  $(5, 1)$  and is perpendicular to  $y = 3$ .

9. Contains  $(5, 1)$  and is parallel to  $y = 3$ .

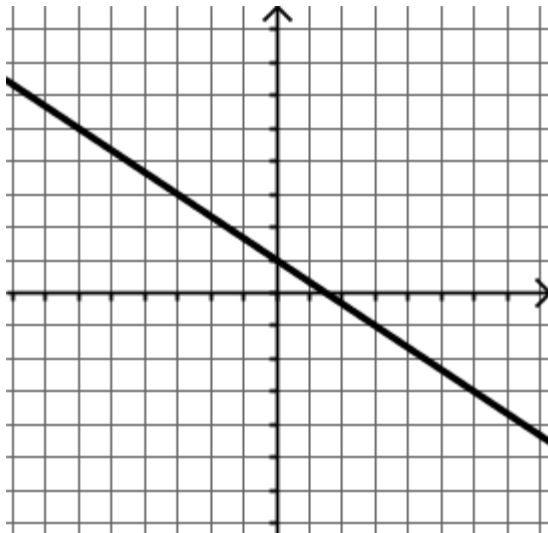
10. Contains  $(-2, -7)$  and is perpendicular to  $x = 4$ .

## Characteristics of Linear Functions (pp. 6 of 8)

### Practice Problems

1.

$x$	$y$



A) Fill in the table of values.

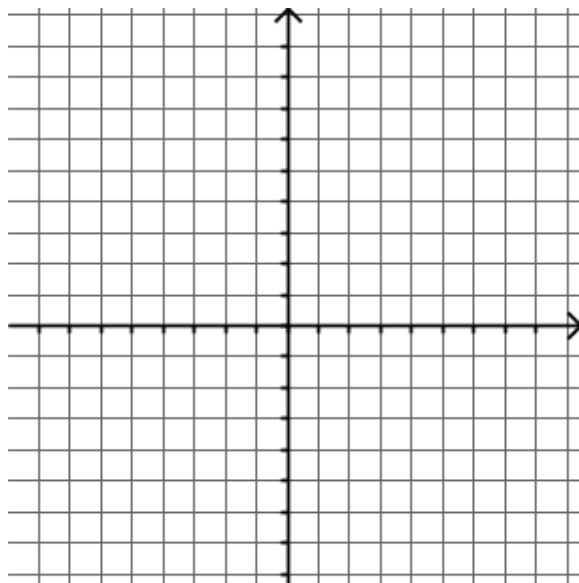
B) Determine the slope.

C) Find the equation for the line.

D) What are the intercepts?

2.

$x$	$y$
-6	3
-3	4
3	6
6	7



A) Graph the function.

B) Determine the slope.

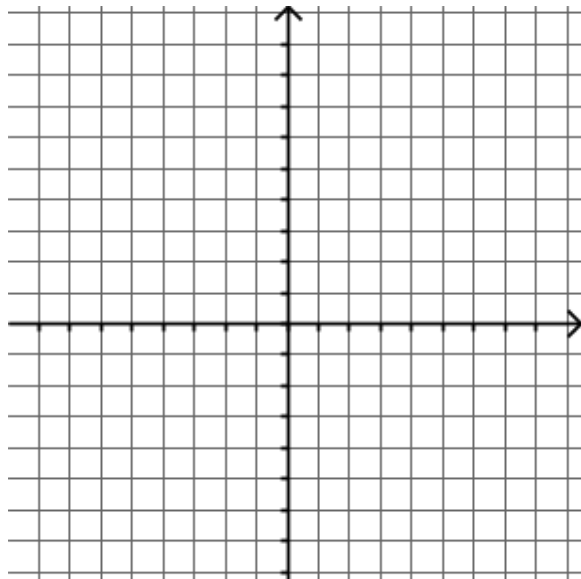
C) Find the equation for the line.

D) What are the intercepts?

## Characteristics of Linear Functions (pp. 7 of 8)

3.

$x$	$y$



Equation:  $y = \frac{3}{2}x - 5$

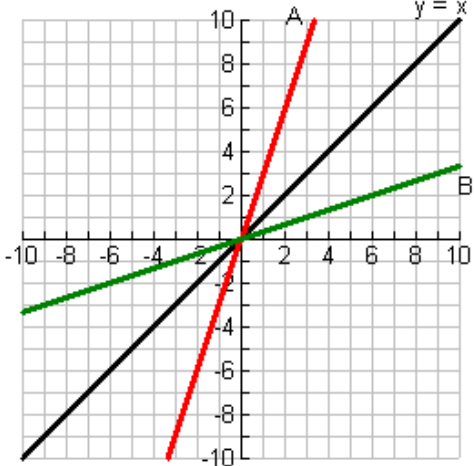
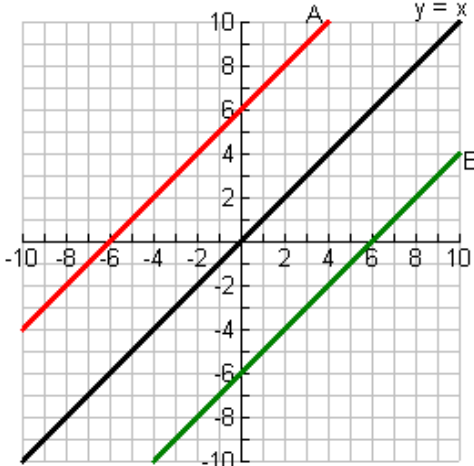
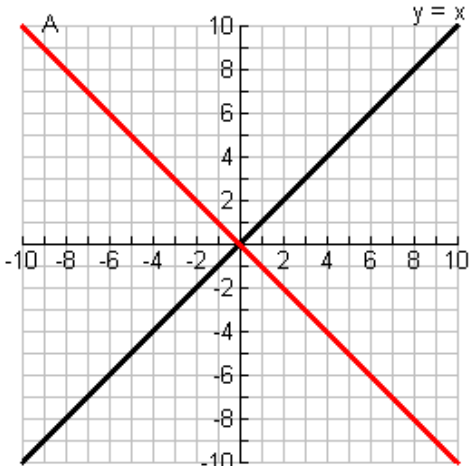
- A) Complete a table of values.
- B) Graph the function.
- C) Write down the slope.
- D) What are the intercepts?

Given information, find the equation of the line in y-intercept and standard forms.

4. The slope of  $-3$  and contains the point  $(0, 3)$
5. The slope of  $\frac{5}{3}$  and contains point  $(-6, -2)$
6. Contains points  $(4, -1)$  and  $(-2, -13)$
7. Contains point  $(-1, 2)$  and is parallel to  $x - 2y = -3$
8. Contains point  $(5, -3)$  and is perpendicular to  $y = 5x - 4$
9. Contains  $(-4, 3)$  and is perpendicular to  $y = 2$ .
10. Contains  $(8, -1)$  and is parallel to  $y = 1$ .
11. Contains  $(-2, -3)$  and is perpendicular to  $x = 2$ .

## Characteristics of Linear Functions (pp. 8 of 8)

12. For each of the linear functions on the graph below, compare it to the linear parent function in terms of vertical shifts and vertical compressions. Identify the parameter that determines the change and determine the function rule.

Graph	Transformations/Changes	Equations
	A)	A)
	B)	B)
	A)	A)
	B)	B)
		



# Characteristics of Linear Functions (pp. 1 of 8) **KEY**

## Parent Function

Linear Parent Function:

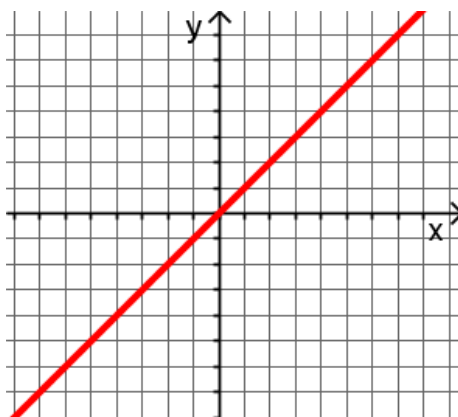
$$y = x$$

Domain:

All real numbers ( $x \in \mathbb{R}$ )

Range:

All real numbers ( $y \in \mathbb{R}$ )



Table

x	y
-3.5	-3.5
-1	-1
0	0
1	1
4.5	4.5
6	6

- What patterns do you observe in the table and graph of the linear parent function?  
 For each point, the  $x$ -values and  $y$ -values are equal. As  $x$  increases,  $y$  increases.  
 Fractional values are possible.

## Review: Characteristics

Characteristics of linear functions (such as slope, intercepts, and equations) have been addressed in previous math courses.

## Forms of Linear Equations

Standard Form:

$$ax + by = c \text{ (No fractions and } a \text{ is positive value)}$$

Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

Slope-Intercept Form:

$$y = mx + b$$

## Slope-Intercept Form

In the slope-intercept form, explain how changes in  $m$  (the slope) and  $b$  (the  $y$ -intercept) affect the graph of the parent function.

Changes caused by $m$	Changes caused by $b$
<ul style="list-style-type: none"> <li>• <math>m = 1</math>, no change in slope of parent function</li> <li>• <math>m &gt; 0</math>, <math>m</math> is positive and the line increases from left to right</li> <li>• <math>m &lt; 0</math>, <math>m</math> is negative and the line reflects in <math>x</math>-axis; decreases from left to right</li> <li>• <math> m  &gt; 1</math>, slope of the parent function increases (vertical stretch)</li> <li>• <math>-1 &lt; m &lt; 1</math>, excluding zero, slope of the parent function decreases (vertical compression)</li> </ul>	<ul style="list-style-type: none"> <li>• <math>b = 0</math>, no change in <math>y</math>-intercept of parent function, stays at <math>(0, 0)</math></li> <li>• <math>b &gt; 0</math>, vertical shift up "<math>b</math>" units</li> <li>• <math>b &lt; 0</math>, vertical shift down "<math>b</math>" units</li> </ul>

## Characteristics of Linear Functions (pp. 2 of 8) **KEY**

### Finding Slope

How would you define slope?

A line's rate of change

A measure of a line's "steepness" or "slant"

Explain how to find slope from each representation.

Graph	Table	Two Points	Equation								
	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>5</td> </tr> <tr> <td>4</td> <td>19</td> </tr> <tr> <td>7</td> <td>40</td> </tr> </tbody> </table> <p>(here, <math>m = 7</math>)</p>	x	y	2	5	4	19	7	40	<p>(-1, 3)</p> <p>and</p> <p>(4, -5)</p> <p>(here, <math>m = -\frac{8}{5}</math>)</p>	<p>A) <math>y = -2x + 4</math> (here, <math>m = -2</math>)</p> <p>B) <math>y = -5 + \frac{2}{3}x</math> (here, <math>m = \frac{2}{3}</math>)</p> <p>C) <math>3x + y = 8</math> (*here, <math>m = -3</math>)</p>
x	y										
2	5										
4	19										
7	40										
<p>Count:</p> $\frac{\text{Rise}}{\text{Run}}$ <p>(here, <math>m = \frac{14}{2} = 7</math>)</p>	<p>Find</p> $\frac{\Delta y}{\Delta x} \text{ or } \frac{\text{change in } y}{\text{change in } x}$	<p>Compute</p> $\frac{y_2 - y_1}{x_2 - x_1}$	<p>Look</p> <p>For the coefficient of the 'x' term (as long as the equation is in <math>y = mx + b</math> form*)</p>								

### Finding Intercepts

What is an intercept?

Point where a line intersects (or crosses) the x-axis or y-axis

Explain how to find a **y-intercept** from each representation.

From the graph:	Determine where the line would cross the <u>y-axis</u>
From a table:	Look for the point <u>(0, y)</u>
From the $y = mx + b$ equation:	Identify the <u>b-value</u>
From any equation:	Plug in <u><math>x = 0</math></u> , then solve for <u>y</u>

Explain how to find an **x-intercept** from each representation.

From the graph:	Determine where the line would cross the <u>x-axis</u>
From a table:	Look for the point <u>(x, 0)</u>
From an equation:	Plug in <u><math>y = 0</math></u> , then solve for <u>x</u>

## Characteristics of Linear Functions (pp. 3 of 8) **KEY**

### Finding Equations

There are several ways to determine the equation of a line, depending on the given information.

If you have...	Then plug values into...
The slope and y-intercept	Slope-intercept Form : $y = mx + b$
The slope, and coordinates for a point	Point-slope Form : $y - y_1 = m(x - x_1)$
Coordinates for two points	$\frac{y_2 - y_1}{x_2 - x_1}$ then $y - y_1 = m(x - x_1)$
A table of data	A calculator's lists (to do a linear regression)

### Special Lines and Slopes

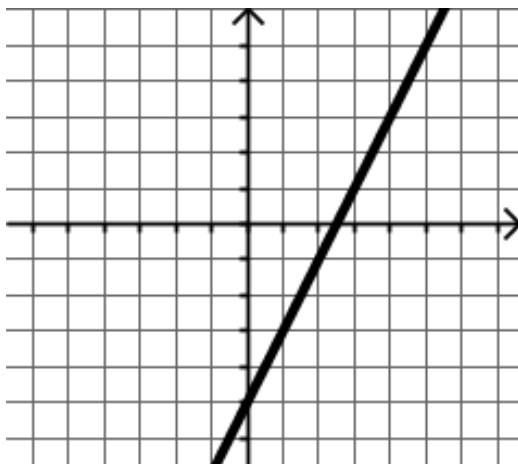
Lines	Slopes	Sample Equation(s)
Horizontal Lines	A horizontal line has a slope of zero	$y = \#$
Vertical Lines	A vertical line has undefined slope	$x = \#$
Parallel Lines	Slope of parallel lines are equal ( $m_1 = m_2$ )	Like $y = 2x + 3$ and $y = 2x - 7$
Perpendicular Lines	Slopes of perpendicular lines are opposite reciprocals ( $-\left(\frac{1}{m_1}\right) = m_2$ )	Like $y = 2x + 3$ and $y = -\frac{1}{2}x - 7$

### Sample Problems

Provide information about each linear function.

1.

x	y
0	-5
7	9
4	3
-2	-9



A) Fill in the table of values.

B) Determine the slope.

$$m = 2$$

C) Find the equation for the line.

$$y = 2x - 5$$

D) What are the intercepts?

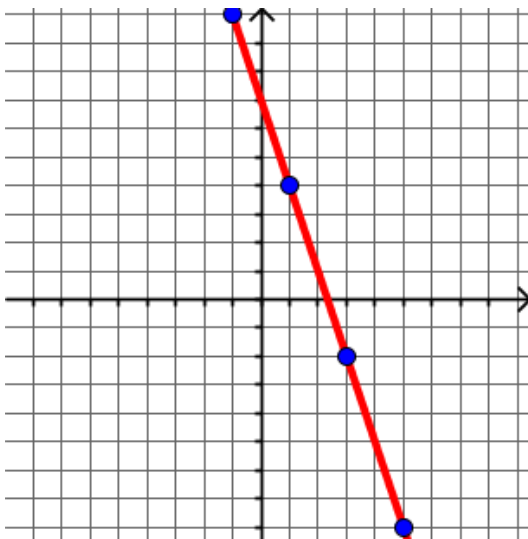
x-intercept: (2.5, 0)

y-intercept: (0, -5)

## Characteristics of Linear Functions (pp. 4 of 8) **KEY**

2.

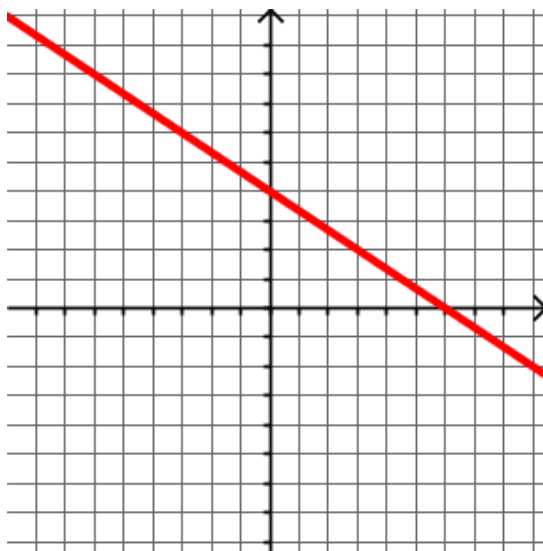
x	y
-1	10
1	4
3	-2
5	-8



- A) Graph the function.  
 B) Determine the slope.  
 $m = -3$
- C) Find the equation for the line.  
 $y = -3x + 7$
- D) What are the intercepts?  
 x-intercept:  $(\frac{7}{3}, 0)$   
 y-intercept:  $(0, 7)$

3.

x	y
-6	8
-3	6
3	2
6	0



- Equation:  $y = -\frac{2}{3}x + 4$
- A) Complete a table of values.  
 B) Graph the function.  
 C) Write down the slope.  
 $m = -\frac{2}{3}$
- D) What are the intercepts?  
 x-intercept:  $(6, 0)$   
 y-intercept:  $(0, 4)$

## Characteristics of Linear Functions (pp. 5 of 8) **KEY**

From the given information, find the equation of the line both in slope-intercept and standard forms.

4. The slope of  $-\frac{1}{2}$  and contains point  $(-2, 5)$

$$y = -\left(\frac{1}{2}\right)x + 4$$

$$x + 2y = 8$$

5. Contains points  $(2, -3)$  and  $(-6, 1)$

$$y = -\left(\frac{1}{2}\right)x - 2$$

$$x + 2y = -4$$

6. Contains point  $(0, 4)$  and is parallel to  $y = 2x - 3$

$$y = 2x + 4$$

$$2x - y = -4$$

7. Contains point  $(-4, 5)$  and is perpendicular to  $2x + 3y = 7$

$$y = \frac{3}{2}x + 11$$

$$3x - 2y = -22$$

8. Contains  $(5, 1)$  and is perpendicular to  $y = 3$ .

$y = 3$  is horizontal, so perpendicular line is vertical with an "x = #" equation  
 $x = 5$  (no slope-intercept form)

9. Contains  $(5, 1)$  and is parallel to  $y = 3$ .

$y = 3$  is horizontal, so parallel line is also horizontal with a "y = #" equation  
 $y = 1$

10. Contains  $(-2, -7)$  and is perpendicular to  $x = 4$ .

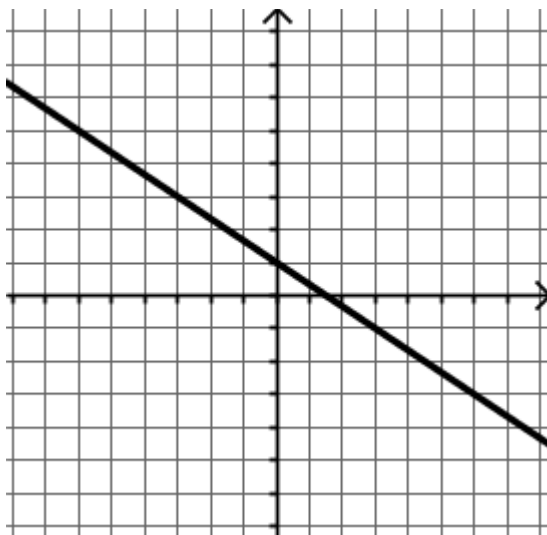
$x = 4$  is vertical, so perpendicular line is horizontal with a "y = #" equation  
 $y = -7$

## Characteristics of Linear Functions (pp. 6 of 8) **KEY**

### Practice Problems

1.

x	y
0	1
-3	3
-6	5
3	-1



A) Fill in the table of values.

B) Determine the slope.

$$m = -\frac{2}{3}$$

C) Find the equation for the line.

$$y = -\frac{2}{3}x + 1$$

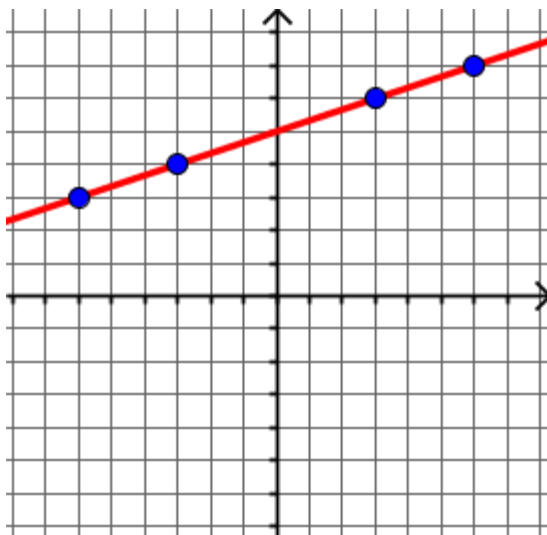
D) What are the intercepts?

x-intercept: (1.5, 0)

y-intercept: (0, 1)

2.

x	y
-6	3
-3	4
3	6
6	7



A) Graph the function.

B) Determine the slope.

$$m = \frac{1}{3}$$

C) Find the equation for the line.

$$y = \frac{1}{3}x + 5$$

D) What are the intercepts?

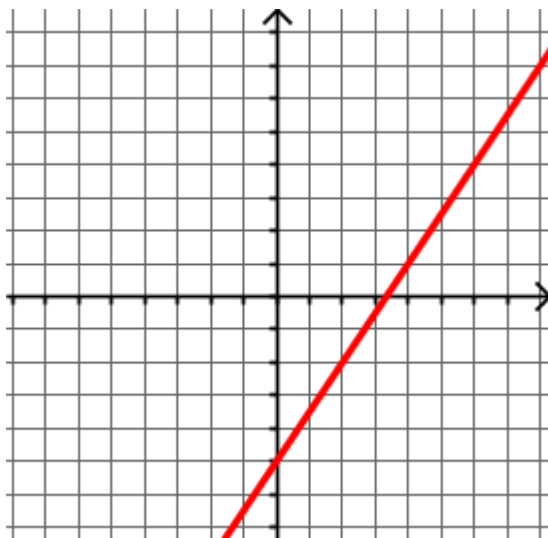
x-intercept: (-15, 0)

y-intercept: (0, 5)

## Characteristics of Linear Functions (pp. 7 of 8) **KEY**

3.

x	y
-2	-8
0	-5
2	-2
4	1



Equation:  $y = \frac{3}{2}x - 5$

- A) Complete a table of values.  
 B) Graph the function.  
 C) Write down the slope.

$$m = \frac{3}{2}$$

- D) What are the intercepts?  
 x-intercept:  $(10/3, 0)$   
 y-intercept:  $(0, 5)$

From the given information, find the equation of the line both in slope-intercept and standard forms.

4. The slope of  $-3$  and contains the point  $(0, 3)$        $y = -3x + 3, 3x + y = 3$

5. The slope of  $\frac{5}{3}$  and contains point  $(-6, -2)$        $y = \frac{5}{3}x + 8, 5x - 3y = -24$

6. Contains points  $(4, -1)$  and  $(-2, -13)$        $y = 2x - 9, 2x - y = 9$

7. Contains point  $(-1, 2)$  and is parallel to  $x - 2y = -3$        $y = \frac{1}{2}x + \frac{5}{2}, x - 2y = -5$

8. Contains point  $(5, -3)$  and is perpendicular to  $y = 5x - 4$        $y = -\frac{1}{5}x - 2, x + 5y = -10$

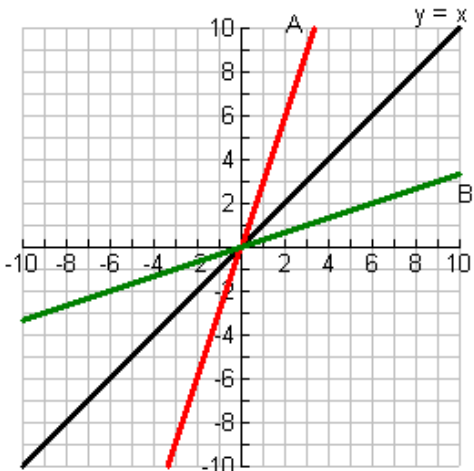
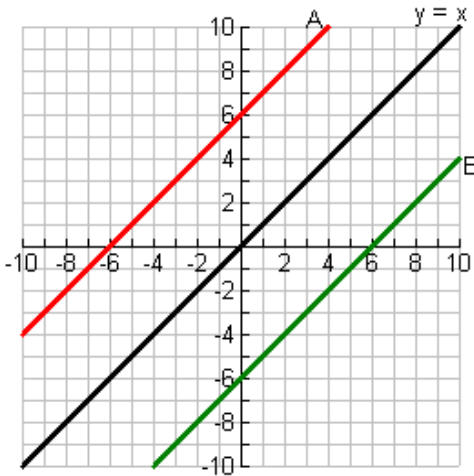
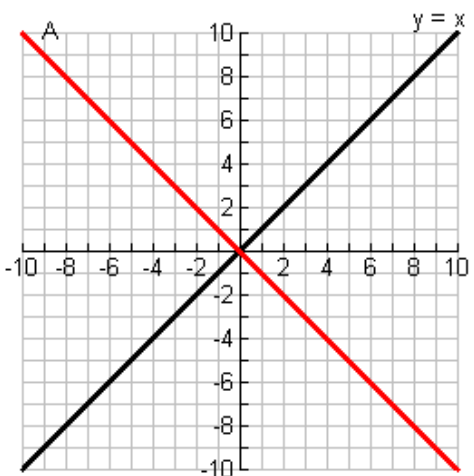
9. Contains  $(-4, 3)$  and is perpendicular to  $y = 2$        $x = -4$

10. Contains  $(8, -1)$  and is parallel to  $y = 1$        $y = -1$

11. Contains  $(-2, -3)$  and is perpendicular to  $x = 2$        $y = -3$

## Characteristics of Linear Functions (pp. 8 of 8) **KEY**

12. For each of the linear functions on the graphs below, compare it to the linear parent function in terms of vertical shifts and vertical compressions. Identify the parameter that determines the change and determine the function rule.

Graph	Transformations/Changes	Equations
	<p>A)  <b>Vertical stretch by a factor of 3, <math>m = 3</math></b></p>	<p>A)  <math>y = 3x</math></p>
	<p>B)  <b>Vertical compression by a factor of <math>\frac{1}{3}</math>,  <math>m = \frac{1}{3}</math></b></p>	<p>B)  <math>y = \frac{1}{3}x</math></p>
	<p>A)  <b>Vertical shift up by 6 units, <math>b = 6</math></b></p>	<p>A)  <math>y = x + 6</math></p>
	<p>B)  <b>Vertical shift down by 6 units, <math>b = -6</math></b></p>	<p>B)  <math>y = x - 6</math></p>
	<p><b>Reflection over the x-axis, <math>m = -1</math></b></p>	<p><math>y = -x</math></p>